**Cheshire College South and West**

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**A-level Maths**

**Summer Work**

Read the examples and complete the questions in each section. The answers are included so that you can check your work.

For additional resources to help you prepare for A-level maths, see

<https://www.pearson.com/en-gb/schools/subject-resources/mathematics/unrivalled-support/support-from-pearson/gcse-maths-transition-to-a-level.html>

**Rules of indices**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* *am* × *an* = *am* + *n*
* 
* (*am*)*n* = *amn*
* *a*0 = 1
*  i.e. the *n*th root of *a*
* 
* 
* The square root of a number produces two solutions, e.g. .

Examples

**Example 1** Evaluate 100

|  |  |
| --- | --- |
| 100 = 1 | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate 

|  |  |
| --- | --- |
| = 3 | Use the rule |

**Example 3** Evaluate 

|  |  |
| --- | --- |
| =  = 9 | **1** Use the rule  **2** Use |

**Example 4** Evaluate 

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use |

**Example 5** Simplify 

|  |  |
| --- | --- |
| = 3*x*3 | 6 ÷ 2 = 3 and use the rule  to give |

**Example 6** Simplify 

|  |  |
| --- | --- |
| = *x*8 − 4 = *x*4 | **1** Use the rule  **2** Use the rule |

**Example 7** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | Use the rule , note that the fraction  remains unchanged |

**Example 8** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use the rule |

Practice

**1** Evaluate.

**a** 140 **b** 30 **c** 50 **d** *x*0

**2** Evaluate.

**a**  **b**  **c**  **d** 

**3** Evaluate.

**a**  **b**  **c**  **d** 

**4** Evaluate.

**a** 5–2 **b** 4–3 **c** 2–5 **d** 6–2

**5** Simplify.

**a**  **b** 

**c**  **d** 

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule *a*0 = 1.

**e**  **f** 

**g**  **h** 

**6** Evaluate.

**a**  **b**  **c** 

**d**  **e**  **f** 

**7** Write the following as a single power of *x*.

**a**  **b**  **c** 

**d**  **e**  **f** 

**8** Write the following without negative or fractional powers.

**a**  **b** *x*0 **c** 

**d**  **e**  **f** 

**9** Write the following in the form *axn*.

**a**  **b**  **c** 

**d**  **e**  **f** 3

# **Extend**

**10** Write as sums of powers of *x*.

**a**  **b**  **c** 

Answers

**1 a** 1 **b** 1 **c** 1 **d** 1

**2 a** 7 **b** 4 **c** 5 **d** 2

**3 a** 125 **b** 32 **c** 343 **d** 8

**4 a**  **b**  **c**  **d** 

**5 a**  **b** 5*x*2

**c** 3*x* **d** 

**e**  **f** *c*–3

**g** 2*x*6 **h** *x*

**6 a**  **b**  **c** 

**d**  **e**  **f** 

**7 a** *x*–1 **b** *x*–7 **c** 

**d**  **e**  **f** 

**8 a**  **b** 1 **c** 

**d**  **e**  **f** 

**9 a**  **b** 2*x*–3 **c** 

**d**  **e**  **f** 3*x*0

**10 a**  **b**  **c** 

**Completing the square**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Completing the square for a quadratic rearranges *ax*2 + *bx* + *c* into the form *p*(*x* + *q*)2 + *r*
* If *a* ≠ 1, then factorise using *a* as a common factor.

Examples

**Example 1** Complete the square for the quadratic expression *x*2 + 6*x* − 2

|  |  |
| --- | --- |
| *x*2 + 6*x* − 2  = (*x* + 3)2 − 9 − 2  = (*x* + 3)2 − 11 | **1** Write *x*2 + *bx* + *c* in the form  **2** Simplify |

**Example 2** Write 2*x*2 − 5*x* + 1 in the form *p*(*x* + *q*)2 + *r*

|  |  |
| --- | --- |
| 2*x*2 − 5*x* + 1  =  =  =  = | **1** Before completing the square write *ax*2 + *bx* + *c* in the form  **2** Now complete the square by writing  in the form  **3** Expand the square brackets – don’t forget to multiply by the factor of 2  **4** Simplify |

Practice

**1** Write the following quadratic expressions in the form (*x* + *p*)2 + *q*

**a** *x*2 + 4*x* + 3 **b** *x*2 – 10*x* – 3

**c** *x*2 – 8*x* **d** *x*2 + 6*x*

**e** *x*2 – 2*x* + 7 **f** *x*2 + 3*x* – 2

**2** Write the following quadratic expressions in the form *p*(*x* + *q*)2 + *r*

**a** 2*x*2 – 8*x* – 16 **b** 4*x*2 – 8*x* – 16

**c** 3*x*2 + 12*x* – 9 **d** 2*x*2 + 6*x* – 8

**3** Complete the square.

**a** 2*x*2 + 3*x* + 6 **b** 3*x*2 – 2*x*

**c** 5*x*2 + 3*x* **d** 3*x*2 + 5*x* + 3

Extend

**4** Write (25*x*2 + 30*x* + 12) in the form (*ax* + *b*)2 + *c*.

Answers

**1 a** (*x* + 2)2 – 1 **b** (*x* – 5)2 – 28

**c** (*x* – 4)2 – 16 **d** (*x* + 3)2 – 9

**e** (*x* – 1)2 + 6 **f** 

**2 a** 2(*x* – 2)2 – 24 **b** 4(*x* – 1)2 – 20

**c** 3(*x* + 2)2 – 21 **d** 

**3 a**  **b** 

**c**  **d** 

**4** (5*x* + 3)2 + 3

**Sketching quadratic graphs**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* The graph of the quadratic function   
  *y* = *ax*2 + *bx* + *c*, where *a* ≠ 0, is a curve   
  called a parabola.

for *a* < 0

for *a* > 0

* Parabolas have a line of symmetry and   
  a shape as shown.
* To sketch the graph of a function, find the points where the graph intersects the axes.
* To find where the curve intersects the *y*-axis substitute *x* = 0 into the function.
* To find where the curve intersects the *x*-axis substitute *y* = 0 into the function.
* At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
* To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

**Example 1** Sketch the graph of *y* = *x*2.

|  |  |
| --- | --- |
|  | The graph of *y* = *x*2 is a parabola.  When *x* = 0, *y* = 0.  *a* = 1 which is greater than zero, so the graph has the shape: |

**Example 2** Sketch the graph of *y* = *x*2 − *x* − 6.

|  |  |
| --- | --- |
| When *x* = 0, *y* = 02 − 0 − 6 = −6  So the graph intersects the *y*-axis at  (0, −6)  When *y* = 0, *x*2 − *x* − 6 = 0  (*x* + 2)(*x* − 3) = 0  *x* = −2 or *x* = 3  So,  the graph intersects the *x*-axis at (−2, 0) and (3, 0)  *x*2 − *x* − 6 =  =  When ,  and , so the turning point is at the point | **1** Find where the graph intersects the *y*-axis by substituting *x* = 0.  **2** Find where the graph intersects the *x*-axis by substituting *y* = 0.  **3** Solve the equation by factorising.  **4** Solve (*x* + 2) = 0 and (*x* − 3) = 0.  **5** *a* = 1 which is greater than zero, so the graph has the shape:  *(continued on next page)*  **6** To find the turning point, complete the square.  **7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero. |

Practice

**1** Sketch the graph of *y* = −*x*2.

**2** Sketch each graph, labelling where the curve crosses the axes.

**a** *y* = (*x* + 2)(*x* − 1) **b** *y* = *x*(*x* − 3) **c** *y* = (*x* + 1)(*x* + 5)

**3** Sketch each graph, labelling where the curve crosses the axes.

**a** *y* = *x*2 − *x* − 6 **b** *y* = *x*2 − 5*x* + 4 **c** *y* = *x*2 – 4

**d** *y* = *x*2 + 4*x* **e** *y* = 9 − *x*2 **f** *y* = *x*2 + 2*x* − 3

**4** Sketch the graph of *y* = 2*x*2 + 5*x* − 3, labelling where the curve crosses the axes.

Extend

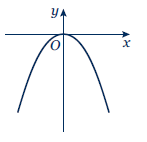
**5** Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

**a** *y* = *x*2 − 5*x* + 6 **b** *y* = −*x*2 + 7*x* − 12 **c** *y* = −*x*2 + 4*x*

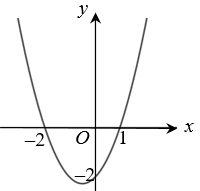
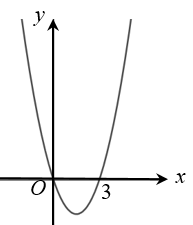
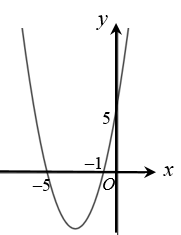
**6** Sketch the graph of *y* = *x*2 + 2*x* + 1. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Answers

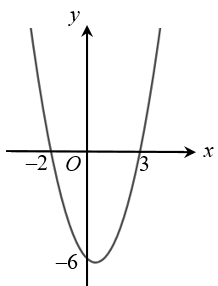
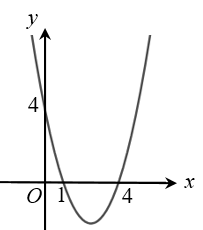
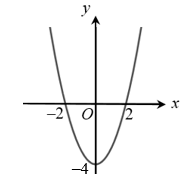
**1**



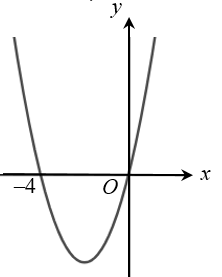
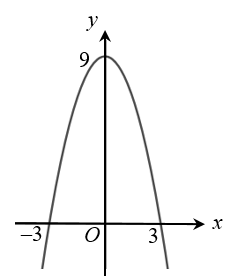
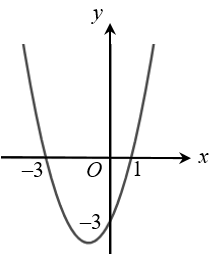
**2 a b c**

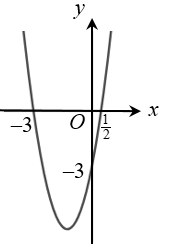
**3 a b c**

**  **

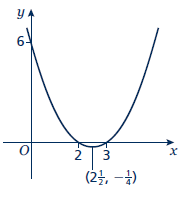
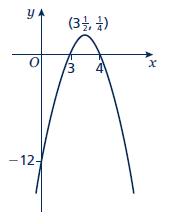
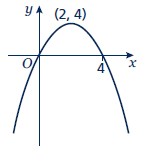
**d e f**

****  

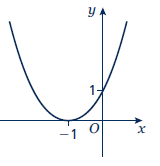
**4**



**5 a b c**

**6**



Line of symmetry at *x* = −1.

**Quadratic inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

* First replace the inequality sign by = and solve the quadratic equation.
* Sketch the graph of the quadratic function.
* Use the graph to find the values which satisfy the quadratic inequality.

Examples

**Example 1** Find the set of values of *x* which satisfy *x*2 + 5*x* + 6 > 0

|  |  |
| --- | --- |
| *x*2 + 5*x* + 6 = 0  (*x* + 3)(*x* + 2) = 0  *x* = −3 or *x* = −2  A graph of a function  Description automatically generated  *x* < −3 or *x* > −2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of  *y* = (*x* + 3)(*x* + 2)  **3** Identify on the graph where  *x*2 + 5*x* + 6 > 0, i.e. where *y* > 0  **4** Write down the values which satisfy the inequality *x*2 + 5*x* + 6 > 0 |

**Example 2** Find the set of values of *x* which satisfy *x*2 − 5*x* ≤ 0

|  |  |
| --- | --- |
| *x*2 − 5*x* = 0  *x*(*x* − 5) = 0  *x* = 0 or *x* = 5  A graph of a function  Description automatically generated  0 ≤ *x* ≤ 5 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = *x*(*x* − 5)  **3** Identify on the graph where  *x*2 − 5*x* ≤ 0, i.e. where *y* ≤ 0  **4** Write down the values which satisfy the inequality *x*2 − 5*x* ≤ 0 |

**Example 3** Find the set of values of *x* which satisfy −*x*2 − 3*x* + 10 ≥ 0

|  |  |
| --- | --- |
| −*x*2 − 3*x* + 10 = 0  (−*x* + 2)(*x* + 5) = 0  *x* = 2 or *x* = −5    −5 ≤ *x* ≤ 2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = (−*x* + 2)(*x* + 5) = 0  **3** Identify on the graph where −*x*2 − 3*x* + 10 ≥ 0, i.e. where *y* ≥ 0  **3** Write down the values which satisfy the inequality −*x*2 − 3*x* + 10 ≥ 0 |

Practice

**1** Find the set of values of *x* for which (*x* + 7)(*x* – 4) ≤ 0

**2** Find the set of values of *x* for which *x*2 – 4*x* – 12 ≥ 0

**3** Find the set of values of *x* for which 2*x*2 –7*x* + 3 < 0

**4** Find the set of values of *x* for which 4*x*2 + 4*x* – 3 > 0

**5** Find the set of values of *x* for which 12 + *x* – *x*2 ≥ 0

Extend

Find the set of values which satisfy the following inequalities.

**6** *x*2 + *x* ≤ 6

**7** *x*(2*x* – 9) < –10

**8** 6*x*2 ≥ 15 + *x*

Answers

**1** –7≤ *x* ≤ 4

**2** *x* ≤ –2 or *x* ≥ 6

**3 **

**4** *x* <  or *x* > 

**5** –3 ≤ *x* ≤ 4

**6** –3 ≤ *x* ≤ 2

**7** 2 < *x* < 2

**8**  or 

**Sketching cubic and reciprocal graphs**

**A LEVEL LINKS**

**Scheme of work:** 1e. Graphs – cubic, quartic and reciprocal

A group of math equations

Description automatically generatedKey points

* The graph of a cubic function, which can be written in the form *y* = *ax*3 + *bx*2 + *cx* + *d*, where *a* ≠ 0, has one of the shapes shown here.

A group of math equations

Description automatically generated

* The graph of a reciprocal function of the form  has one of the shapes shown here.
* To sketch the graph of a function, find the points where the graph intersects the axes.
* To find where the curve intersects the *y*-axis substitute *x* = 0 into the function.
* To find where the curve intersects the *x*-axis substitute *y* = 0 into the function.
* Where appropriate, mark and label the asymptotes on the graph.
* Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of  are the two axes (the lines *y* = 0 and *x* = 0).
* At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
* A double root is when two of the solutions are equal. For example (*x* – 3)2(*x* + 2) has a double root at *x* = 3.
* When there is a double root, this is one of the turning points of a cubic function.

Examples

**Example 1** Sketch the graph of *y* = (*x* − 3)(*x* − 1)(*x* + 2)

|  |  |
| --- | --- |
| To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape. | |
| When *x* = 0, *y* = (0 − 3)(0 − 1)(0 + 2)  = (−3) × (−1) × 2 = 6  The graph intersects the *y*-axis at (0, 6)  When *y* = 0, (*x* − 3)(*x* − 1)(*x* + 2) = 0  So *x* = 3, *x* = 1 or *x* = −2  The graph intersects the *x*-axis at   (−2, 0), (1, 0) and (3, 0)  A graph of a function  Description automatically generated | **1** Find where the graph intersects the axes by substituting *x* = 0 and *y* = 0.  Make sure you get the coordinates the right way around, (*x*, *y*).  **2** Solve the equation by solving  *x* − 3 = 0, *x* − 1 = 0 and *x* + 2 = 0  **3** Sketch the graph.  *a* = 1 > 0 so the graph has the shape:  A graph of a function  Description automatically generated |

**Example 2** Sketch the graph of *y* = (*x* + 2)2(*x* − 1)

|  |  |
| --- | --- |
| To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape. | |
| When *x* = 0, *y* = (0 + 2)2(0 − 1)  = 22 × (−1) = −4  The graph intersects the *y*-axis at (0, −4)  When *y* = 0, (*x* + 2)2(*x* − 1) = 0  So *x* = −2 or *x* =1  (−2, 0) is a turning point as *x* = −2 is a double root. The graph crosses the *x*-axis at (1, 0)  A graph of a function  Description automatically generated | **1** Find where the graph intersects the axes by substituting *x* = 0 and *y* = 0.  **2** Solve the equation by solving  *x* + 2 = 0 and *x* − 1 = 0  **3** *a* = 1 > 0 so the graph has the shape:  A graph of a function  Description automatically generated |

Practice

**1** Here are six equations.

**Hint**

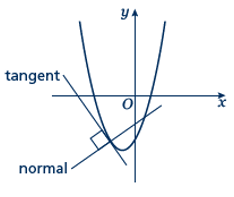
Find where each of the cubic equations cross the *y*-axis.

**A**  **B** *y* = *x*2 + 3*x* – 10 **C** *y* = *x*3 + 3*x*2

**D** *y* = 1 – 3*x*2 – *x*3 **E** *y* = *x*3 – 3*x*2 – 1 **F** *x* + *y* = 5

A graph of a function

Description automatically generated Here are six graphs.

**A graph of normal and normal

Description automatically generated i ii iii**

**A graph of a function

Description automatically generated**

**A graph of a function

Description automatically generatedA graph of a function

Description automatically generated iv v vi**

**a** Match each graph to its equation.

**b** Copy the graphs ii, iv and vi and draw the tangent and normal each at point *P*.

Sketch the following graphs

**2**  *y* = 2*x*3 **3** *y* = *x*(*x* – 2)(*x* + 2)

**4** *y* = (*x* + 1)(*x* + 4)(*x* – 3) **5** *y* = (*x* + 1)(*x* – 2)(1 – *x*)

**6** *y* = (*x* – 3)2(*x* + 1) **7** *y* = (*x* – 1)2(*x* – 2)

**8** *y* =  **9** *y* = 

**Hint:** Look at the shape of *y* =  in the second key point.

Extend

**10** Sketch the graph of  **11** Sketch the graph of 

Answers

**1****a** i – C

ii – E

iii – B

iv – A

v – F

vi – D

**A diagram of a function

Description automatically generatedA graph of a function

Description automatically generated b ii iv**

**A diagram of a function

Description automatically generated**

**vi**

**A graph of a function

Description automatically generatedA graph of a function

Description automatically generated2 3**

**A graph of a function

Description automatically generated**

**A graph of a function

Description automatically generated4 5**

**A graph of a function

Description automatically generatedA graph of a function

Description automatically generated6 7**

**A graph of a function

Description automatically generated8 9**

**A graph of a function

Description automatically generatedA graph of function and numbers

Description automatically generated10 11**

**Straight line graphs**

**A LEVEL LINKS**

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

A math equations on a white background

Description automatically generatedKey points

* A straight line has the equation *y* = *mx* + *c*, where *m* is the gradient and *c* is the *y*-intercept (where *x* = 0).
* The equation of a straight line can be written in the form *ax* + *by* + *c* = 0, where *a*, *b* and *c* are integers.
* When given the coordinates (*x*1, *y*1) and (*x*2, *y*2) of two points on a line the gradient is calculated using the formula 

Examples

**Example 1** A straight line has gradient  and *y*-intercept 3.  
Write the equation of the line in the form *ax* + *by* + *c* = 0.

|  |  |
| --- | --- |
| *m* =  and *c* = 3  So *y* = *x* + 3  *x* + *y* – 3 = 0  *x* + 2*y* − 6 = 0 | **1** A straight line has equation *y*= *mx*+ *c*. Substitute the gradient and *y*-intercept given in the question into thisequation.  **2** Rearrange the equation so all the terms are on one side and 0 is on  the other side.  **3** Multiply both sides by 2 to eliminate the denominator. |

**Example 2** Find the gradient and the *y*-intercept of the line with the equation 3*y* − 2*x* + 4 = 0.

|  |  |
| --- | --- |
| 3*y* − 2*x* + 4 = 0  3*y* = 2*x* − 4    Gradient = *m* =  *y*-intercept = *c* = | **1** Make *y* the subject of the equation.  **2** Divide all the terms by three to get the equation in the form *y* = …  **3** In the form *y* = *mx* + *c*, the gradient is *m* and the *y*-intercept is *c*. |

**Example 3** Find the equation of the line which passes through the point (5, 13) and has gradient 3.

|  |  |
| --- | --- |
| *m* = 3  *y* = 3*x* + *c*  13 = 3 × 5 + *c*  13 = 15 + *c*  *c* = −2  *y* = 3*x* − 2 | **1** Substitute the gradient given in the question into the equation of a straight line *y* = *mx* + *c*.  **2** Substitute the coordinates *x* = 5 and *y* = 13 into the equation.  **3** Simplify and solve the equation.  **4** Substitute *c* = −2 into the equation *y*= 3*x*+ *c* |

**Example 4** Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

|  |  |
| --- | --- |
| , ,  and        *c* = 3 | **1** Substitute the coordinates into the equation  to work out the gradient of the line.  **2** Substitute the gradient into the equation of a straight line *y*= *mx*+ *c*.  **3** Substitute the coordinates of either point into the equation.  **4** Simplify and solve the equation.  **5** Substitute *c* = 3 into the equation |

Practice

**1** Find the gradient and the *y*-intercept of the following equations.

**a** *y* = 3*x* + 5 **b** *y* = *x* – 7

**Hint**

Rearrange the equations to the form *y* = *mx* + *c*

**c** 2*y* = 4*x* – 3 **d** *x* + *y* = 5

**e** 2*x* – 3*y* – 7 = 0 **f** 5*x* + *y* – 4 = 0

**2** Copy and complete the table, giving the equation of the line in the form *y* = *mx* + *c*.

|  |  |  |
| --- | --- | --- |
| **Gradient** | ***y*-intercept** | **Equation of the line** |
| 5 | 0 |  |
| –3 | 2 |  |
| 4 | –7 |  |

**3** Find, in the form *ax* + *by* + *c* = 0 where *a*, *b* and *c* are integers, an equation for each of the lines with the following gradients and *y*-intercepts.

**a** gradient , *y*-intercept –7 **b** gradient 2, *y*-intercept 0

**c** gradient , *y*-intercept 4 **d** gradient –1.2, *y*-intercept –2

**4** Write an equation for the line which passes though the point (2, 5) and has gradient 4.

**5** Write an equation for the line which passes through the point (6, 3) and has gradient 

**6** Write an equation for the line passing through each of the following pairs of points.

**a** (4, 5), (10, 17) **b** (0, 6), (–4, 8)

**c** (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Extend

**7** The equation of a line is 2*y* + 3*x* – 6 = 0.  
Write as much information as possible about this line.

Answers

**1** **a** *m* = 3, *c* = 5 **b** *m* = , *c* = –7

**c** *m* = 2, *c* =  **d** *m* = –1, *c* = 5

**e** *m* = , *c* = or –2 **f** *m* = –5, *c* = 4

**2**

|  |  |  |
| --- | --- | --- |
| **Gradient** | ***y*-intercept** | **Equation of the line** |
| 5 | 0 | *y* = 5*x* |
| –3 | 2 | *y* = –3*x* + 2 |
| 4 | –7 | *y* = 4*x* –7 |

**3 a** *x* + 2*y* + 14 = 0 **b** 2*x* – *y* = 0

**c** 2*x* – 3*y* + 12 = 0 **d** 6*x* + 5*y* + 10 = 0

**4** *y* = 4*x* – 3

**5** *y* = *x* + 7

**6 a** *y* = 2*x* – 3 **b** *y* = *x* + 6

**c** *y* = 5*x* –2 **d** *y* = –3*x* + 19

**7** , the gradient is  and the *y*-intercept is 3.  
The line intercepts the axes at (0, 3) and (2, 0).  
Students may sketch the line or give coordinates that lie on the line such as  or .

**Trigonometry in right-angled triangles**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

A diagram of a triangle

Description automatically generatedKey points

* In a right-angled triangle:
* the side opposite the right angle is called the hypotenuse
* the side opposite the angle *θ* is called the opposite
* the side next to the angle *θ* is called the adjacent.
* In a right-angled triangle:
  + the ratio of the opposite side to the hypotenuse is the sine of angle *θ*, 
  + the ratio of the adjacent side to the hypotenuse is the cosine of angle *θ*, 
  + the ratio of the opposite side to the adjacent side is the tangent of angle *θ*, 
* If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: sin−1, cos−1, tan−1.
* The sine, cosine and tangent of some angles may be written exactly.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **0** | **30°** | **45°** | **60°** | **90°** |
| **sin** | 0 |  |  |  | 1 |
| **cos** | 1 |  |  |  | 0 |
| **tan** | 0 |  | 1 |  |  |

A triangle with text and numbers

Description automatically generatedExamples

**Example 1** Calculate the length of side *x*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with text on it  Description automatically generated        *x* = 6.620 267 5...  *x* = 6.62 cm | **1** Always start by labelling the sides.  **2** You are given the adjacent and the hypotenuse so use the cosine ratio.  **3** Substitute the sides and angle into the cosine ratio.  **4** Rearrange to make *x* the subject.  **5** Use your calculator to work out  6 ÷ cos 25°.  **6** Round your answer to 3 significant figures and write the units in your answer. |

A drawing of a triangle with a circle and a circle

Description automatically generated

**Example 2** Calculate the size of angle *x*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with text on it  Description automatically generated with medium confidence      *x* = tan–1  *x* = 33.690 067 5...  *x* = 33.7° | **1** Always start by labelling the sides.  **2** You are given the opposite and the adjacent so use the tangent ratio.  **3** Substitute the sides and angle into the tangent ratio.  **4** Use tan−1 to find the angle.  **5** Use your calculator to work out  tan–1(3 ÷ 4.5).  **6** Round your answer to 3 significant figures and write the units in your answer. |

A black triangle with white text

Description automatically generated**Example 3** Calculate the exact size of angle *x*.

|  |  |
| --- | --- |
| A triangle with text on it  Description automatically generated      *x* = 30° | **1** Always start by labelling the sides.  **2** You are given the opposite and the adjacent so use the tangent ratio.  **3** Substitute the sides and angle into the tangent ratio.  **4** Use the table from the key points to find the angle. |

Practice

**1** Calculate the length of the unknown side in each triangle.  
 Give your answers correct to 3 significant figures.

A triangle with text on it

Description automatically generatedA black triangle with white text

Description automatically generated **a b**

A black line drawing of a triangle

Description automatically generatedA black triangle with black text

Description automatically generated **c d**

A black and white triangle with text

Description automatically generatedA black triangle with white text

Description automatically generated **e f**

A black triangle with text and numbers

Description automatically generated**2** Calculate the size of angle *x* in each triangle.  
 Give your answers correct to 1 decimal place.

A triangle with text on it

Description automatically generated **a b**

A black triangle with black lines

Description automatically generated

A black and white drawing of a triangle

Description automatically generated **c d**

A triangle with numbers and a triangle in the middle

Description automatically generated**3** Work out the height of the isosceles triangle.  
 Give your answer correct to 3 significant figures.

**Hint:**

Split the triangle into two right-angled triangles.

A triangle with a square in the middle

Description automatically generated with medium confidence**4** Calculate the size of angle *θ*.  
 Give your answer correct to 1 decimal place.

**Hint:**

First work out the length of the common side to both triangles, leaving your answer in surd form.

A triangle with square and square

Description automatically generatedA black triangle with white text

Description automatically generated**5** Find the exact value of *x* in each triangle.

**a b**

A triangle with text and numbers

Description automatically generated

A black line with white text

Description automatically generated **c d**

**The cosine rule**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:**Pure Year 1, 9.1 The cosine rule

A triangle with letters and numbers

Description automatically generatedKey points

* *a* is the side opposite angle A.

*b* is the side opposite angle B.

*c* is the side opposite angle C.

* You can use the cosine rule to find the length of a side when two sides and the included angle are given.
* To calculate an unknown side use the formula .
* Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
* To calculate an unknown angle use the formula .

A triangle with a circle and a circle with letters

Description automatically generatedExamples

**Example 4** Work out the length of side *w*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with text on it  Description automatically generated      *w*2 = 33.804 040 51...  *w* =  *w* = 5.81 cm | **1** Always start by labelling the angles and sides.  **2** Write the cosine rule to find the side.  **3** Substitute the values *a*, *b* and *A* into the formula.  **4** Use a calculator to find *w*2 and  then *w*.  **5** Round your final answer to 3 significant figures and write the units in your answer. |

A triangle with a circle and a circle with the same size

Description automatically generated with medium confidence**Example 5** Work out the size of angle *θ*.  
 Give your answer correct to 1 decimal place.

|  |  |
| --- | --- |
| A triangle with numbers and a circle  Description automatically generated        *θ* = 122.878 349...  *θ* = 122.9° | **1** Always start by labelling the angles and sides.  **2** Write the cosine rule to find the angle.  **3** Substitute the values *a*, *b* and *c* into the formula.  **4** Use cos−1 to find the angle.  **5** Use your calculator to work out  cos–1(–76 ÷ 140).  **6** Round your answer to 1 decimal place and write the units in your answer. |

Practice

**6** Work out the length of the unknown side in each triangle.  
 Give your answers correct to 3 significant figures.

A black triangle with white text

Description automatically generated **a b**

A black and white triangle with a circle and black text

Description automatically generatedA black triangle with white text

Description automatically generated

A triangle with text on it

Description automatically generated **c d**

A triangle with numbers and a line

Description automatically generated**7** Calculate the angles labelled *θ* in each triangle.  
 Give your answer correct to 1 decimal place.

A black triangle with white text

Description automatically generated **a b**

A triangle with text on it

Description automatically generatedA drawing of a triangle with numbers and a circle

Description automatically generated **c d**

A triangle with a number of degrees

Description automatically generated with medium confidence**8 a** Work out the length of WY.  
 Give your answer correct to   
 3 significant figures.

**b** Work out the size of angle WXY.  
 Give your answer correct to   
 1 decimal place.

**The sine rule**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:**Pure Year 1, 9.2 The sine rule

A triangle with letters and numbers

Description automatically generatedKey points

* *a* is the side opposite angle A.  
  *b* is the side opposite angle B.  
  *c* is the side opposite angle C.
* You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
* To calculate an unknown side use the formula .
* Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
* To calculate an unknown angle use the formula .

A triangle with numbers and a circle

Description automatically generatedExamples

**Example 6** Work out the length of side *x*.  
 Give your answer correct to 3 significant figures.

|  |  |
| --- | --- |
| A triangle with numbers and a few circles  Description automatically generated with medium confidence        *x* = 6.09 cm | **1** Always start by labelling the angles and sides.  **2** Write the sine rule to find the side.  **3** Substitute the values *a*, *b*, *A* and *B* into the formula.  **4** Rearrange to make *x* the subject.  **5** Round your answer to 3 significant figures and write the units in your answer. |

A triangle with a circle and numbers

Description automatically generated**Example 7** Work out the size of angle *θ*.  
 Give your answer correct to 1 decimal place.

|  |  |
| --- | --- |
| A triangle with a circle and a circle in the center  Description automatically generated        *θ* = 27.2° | **1** Always start by labelling the angles and sides.  **2** Write the sine rule to find the angle.  **3** Substitute the values *a*, *b*, *A* and *B* into the formula.  **4** Rearrange to make sin *θ* the subject.  **5** Use sin−1 to find the angle. Round your answer to 1 decimal place and write the units in your answer. |

Practice

**9** Find the length of the unknown side in each triangle.  
 Give your answers correct to 3 significant figures.

A triangle with numbers and a few circles

Description automatically generated with medium confidenceA triangle with numbers and a few letters

Description automatically generated with medium confidence

**a b**

A triangle with numbers and circles

Description automatically generated

A black triangle with white text

Description automatically generated **c d**

**10** Calculate the angles labelled *θ* in each triangle.  
 Give your answer correct to 1 decimal place.

A black and white image of a triangle with white text

Description automatically generatedA triangle with a circle and circles

Description automatically generated

**a b**

A triangle with a circle and a circle in the middle

Description automatically generated

A triangle with numbers and a circle

Description automatically generated **c d**

A black and white triangle with numbers and a circle

Description automatically generated**11** **a** Work out the length of QS.  
 Give your answer correct to 3 significant figures.

**b** Work out the size of angle RQS.  
 Give your answer correct to 1 decimal place.

**Areas of triangles**

**A LEVEL LINKS**

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:**Pure Year 1, 9.3 Areas of triangles

A triangle with text on it

Description automatically generatedKey points

* *a* is the side opposite angle A.  
  *b* is the side opposite angle B.  
  *c* is the side opposite angle C.
* The area of the triangle is .

A triangle with numbers and a circle

Description automatically generatedExamples

**Example 8** Find the area of the triangle.

|  |  |
| --- | --- |
| A triangle with text on it  Description automatically generated  Area =  Area =  Area = 19.805 361...  Area = 19.8 cm2 | **1** Always start by labelling the sides and angles of the triangle.  **2** State the formula for the area of a triangle.  **3** Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle.  **4** Use a calculator to find the area.  **5** Round your answer to 3 significant figures and write the units in your answer. |

Practice

**12** Work out the area of each triangle.  
 Give your answers correct to 3 significant figures.

A black triangle with white text

Description automatically generatedA black triangle with white text

Description automatically generated **a b**

A triangle with numbers and a circle

Description automatically generated **c**

A triangle with text on it

Description automatically generated

**13** The area of triangle XYZ is 13.3 cm2.  
 Work out the length of XZ.

**Hint:**

Rearrange the formula to make a side the subject.

Extend

**Hint:**

For each one, decide whether to use the cosine or sine rule.

**14** Find the size of each lettered angle or side.  
 Give your answers correct to 3 significant figures.

A black triangle with black text

Description automatically generatedA triangle with numbers and a circle

Description automatically generated with medium confidence **a b**

A black line with black text

Description automatically generatedA triangle with a circle and numbers

Description automatically generated with medium confidence **c d**

A triangle with text on it

Description automatically generated**15** The area of triangle ABC is 86.7 cm2.  
 Work out the length of BC.  
 Give your answer correct to 3 significant figures.

Answers

**1 a** 6.49 cm **b** 6.93 cm **c** 2.80 cm   
 **d** 74.3 mm **e** 7.39 cm **f** 6.07 cm

**2 a** 36.9° **b** 57.1° **c** 47.0° **d** 38.7°

**3** 5.71 cm

**4** 20.4°

**5 a** 45° **b** 1 cm **c** 30° **d**  cm

**6 a** 6.46 cm **b** 9.26 cm **c** 70.8 mm **d** 9.70 cm

**7 a** 22.2° **b** 52.9° **c** 122.9° **d** 93.6°

**8 a** 13.7 cm **b** 76.0°

**9 a** 4.33 cm **b** 15.0 cm **c** 45.2 mm **d** 6.39 cm

**10 a** 42.8° **b** 52.8° **c** 53.6° **d** 28.2°

**11 a** 8.13 cm **b** 32.3°

**12 a** 18.1 cm2 **b** 18.7 cm2 **c** 693 mm2

**13** 5.10 cm

**14 a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

**15** 15.3 cm

**Rearranging equations**

**A LEVEL LINKS**

**Scheme of work:** 6a. Definition, differentiating polynomials, second derivatives

**Textbook:**Pure Year 1, 12.1 Gradients of curves

Key points

* To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
* You may need to factorise the terms containing the new subject.

Examples

**Example 1** Make *t* the subject of the formula *v* = *u* + *at*.

|  |  |
| --- | --- |
| *v* = *u* + *at*  *v* − *u* = *at* | **1** Get the terms containing *t* on one side and everything else on the other side.  **2** Divide throughout by *a*. |

**Example 2** Make *t* the subject of the formula *r* = 2*t* − *πt*.

|  |  |
| --- | --- |
| *r* = 2*t* − *πt*  *r* = *t*(2 − *π*) | **1** All the terms containing *t* are already on one side and everything else is on the other side.  **2** Factorise as *t* is a common factor.  **3** Divide throughout by 2 − *π*. |

**Example 3** Make *t* the subject of the formula .

|  |  |
| --- | --- |
| 2*t* + 2*r* = 15*t*  2*r* = 13*t* | **1** Remove the fractions first by multiplying throughout by 10.  **2** Get the terms containing *t* on one side and everything else on the other side and simplify.  **3** Divide throughout by 13. |

**Example 4** Make *t* the subject of the formula .

|  |  |
| --- | --- |
| *r*(*t* − 1) = 3*t* + 5  *rt* − *r* = 3*t* + 5  *rt* − 3*t* = 5 + *r*  *t*(*r* − 3) = 5 + *r* | **1** Remove the fraction first by multiplying throughout by *t* − 1.  **2** Expand the brackets.  **3** Get the terms containing *t* on one side and everything else on the other side.  **4** Factorise the LHS as *t* is a common factor.  **5** Divide throughout by *r* − 3. |

Practice

Change the subject of each formula to the letter given in the brackets.

**1** *C* = *πd*  [*d*]**2** *P* = 2*l* + 2*w* [*w*] **3** *D = * [*T*]

**4** ** [*t*] **5** *u* = *at* – *t* [*t*] **6** *V* = *ax* + 4*x* [*x*]

**7** ** [*y*] **8**  [*a*] **9**  [*d*]

**10**  [*g*] **11** *e*(9 + *x*) = 2*e* + 1 [*e*] **12**  [*x*]

**13** Make *r* the subject of the following formulae.

**a** *A* = *πr*2 **b**  **c** *P* = *πr* + 2*r* **d** 

**14** Make *x* the subject of the following formulae.

**a  b **

**15** Make sin *B* the subject of the formula 

**16** Make cos *B* the subject of the formula *b*2 = *a*2 + *c*2 – 2*ac* cos *B*.

Extend

**17** Make *x* the subject of the following equations.

**a**  **b** 

Answers

**1** *d* =  **2**  **3** 

**4**  **5**  **6** 

**7** *y* = 2 + 3*x* **8**  **9** 

**10**  **11**  **12** 

**13 a**  **b** 

**c**  **d** 

**14 a**  **b** 

**15** 

**16** 

**17 a**  **b** 